Numeric Response Questions

Vectors

Q.1 If $\hat{\imath} - \hat{\jmath} + 2\hat{k}, 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ are position vectors of vertices of a triangle, if its area is $\sqrt{k}$ then find k.

Q.2 A particle acted on by constant forces $4\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - \hat{k}$ is displaced from the point $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ to the point $5\hat{\imath} + 4\hat{\jmath} + \hat{k}$. Then find the total work done by forces.

Q.3 Let $\vec{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|\vec{c} - \vec{a}\ | = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $30°$, then find value of $|(\vec{a} \times \vec{b}) \times \vec{c}|$.

Q.4 If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then find $|\vec{a} - \vec{b}|$.

Q.5 ABCD is a quadrilateral with $\overrightarrow{AB} = \vec{a}, \overrightarrow{AD} = \vec{b}$ and $\overrightarrow{AC} = 2\vec{a} + 3\vec{b}$. If its area is α times the area of the parallelogram gm with AB, AD as adjacent sides, then find the value of α,

Q.6 Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 10\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$ where A and C are non-collinear points, Let p denote the area of the quadrilateral OABC, and let q denote the area of the parallelogram with OA and OC as adjacent sides, If $p = kq$, then find value of k.

Q.7 If vectors $\overrightarrow{AB} = -3\hat{\imath} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ are the sides of a $\triangle ABC$, if the length of the median through A is $\sqrt{k}$ then find k.

Q.8 If $|\vec{a}| = |\vec{b}| = 1$ and $|\vec{a} + \vec{b}| = \sqrt{3}$, then find the value of $(3\vec{a} - 4\vec{b}), (2\vec{a} + 5\vec{b})$.

Q.9 Find the area of the parallelogram whose diagonals are $\vec{a} - \vec{b}$ and $3\vec{a} + \vec{b}$, where $|\vec{a}| = 2|\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$.

Q.10 Let $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$ and $\vec{a} \perp (\vec{b} + \vec{c}), \vec{b} \perp (\vec{c} + \vec{a})$ and $\vec{c} \perp (\vec{a} + \vec{b})$, If the value of $|\vec{a} + \vec{b} + \vec{c}|$ is $\sqrt{k}$ then find k, Q.11 If vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + a\hat{\jmath} + 5\hat{k}$ are coplanar, then find value of ' a '.

Q.12 If the vectors $\vec{a} + \vec{b} - \lambda\vec{c}$, $3\vec{a} - 2\vec{b} + 4\vec{c}$, $3\vec{a} - 7\vec{b} + 14\vec{c}$ are linearly dependent ($\vec{a}, \vec{b}, \vec{c}$ are non zero noncoplanar) then find the value of λ.

Q.13 If $\vec{u}, \vec{v}, \vec{w}$ are vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$, $|\vec{u}| = 3$, $|\vec{v}| = 4$ and $|\vec{w}| = 5$, then find value of $\vec{u} \cdot \vec{v} + \vec{v}, \vec{w} + \vec{w}, \vec{u}$.

Q.14 Let $|\vec{a}| = 1$ and $|\vec{b}| = 3$. If $\vec{a} + \lambda\vec{b}$ and $\vec{a} - \lambda\vec{b}$ are mutually perpendicular vectors, then find value of $|\lambda|$,

Q.15 Find the volume of parallelopiped whose coterminus edges are represented by $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$. $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

ANSWER KEY

1. 13.00	**2.** 40.00	**3.** 1.50	**4.** 5.00	**5.** 2.50	**6.** 6.00	**7.** 18.00
8. – 10.50	**9.** 2.00	**10.** 14.00	**11.** –4.00	**12.** 2.00	**13.** – 25.00	**14.** 0.33
15. 7.00						

Hints & Solutions

1. $\overrightarrow{AB} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\overrightarrow{AC} = 2\hat{i}$

Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \sqrt{13}$

2. $\vec{F} = 7\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{d} = (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{d} = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore\ W = \vec{F}.\vec{d} = 40$

3. $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

$|\vec{c} - \vec{a}| = 2\sqrt{2}$

$|\vec{c} - \vec{a}|^2 = 8$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{c} = 8$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c} = 8$

$|\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$\therefore\ (|\vec{c}| - 1)^2 = 0$

$|\vec{c}| = 1$

$\therefore |(\vec{a} \times \vec{b}) \times \vec{c}|$

$= |\vec{a} \times \vec{b}||\vec{c}|\ \sin 30^\circ = \frac{3}{2}$

4. $|\vec{a}| = 3,\ |\vec{b}| = 4,\ |\vec{a} + \vec{b}| = 5$

$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos\theta = 25$

$9 + 16 + 2 \times 3 \times 4 \cos\theta = 25$

$\cos\theta = 0 \quad \Rightarrow\ \theta = \pi/2$

$|\vec{a} - \vec{b}| = \sqrt{9 + 16 - 2 \times 3 \times 4(0)} = 5$

5. Area of quadrilateral ABCD

= area of ΔABC + area of ΔACD

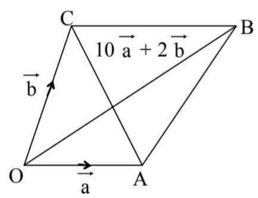
$= \frac{1}{2}[\vec{a} \times (2\vec{a} + 3\vec{b})] + \frac{1}{2}[(2\vec{a} + 3\vec{b}) \times \vec{b}]$

$= \frac{1}{2}(3\vec{a} \times \vec{b} + 2\vec{a} \times \vec{b}) = \frac{5}{2}(\vec{a} \times \vec{b})$

$= \frac{5}{2}$ area of parallelogram ABCD

$\Rightarrow \alpha = \frac{5}{2}$

6.



$p = \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{OB}|$

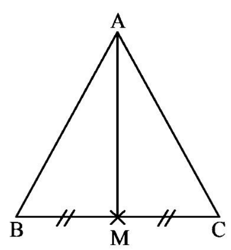
$p = \frac{1}{2}|(\vec{b} - \vec{a}) \times (10\vec{a} + 2\vec{b})|$

$p = \frac{1}{2}|10(\vec{b} \times \vec{a}) - 2(\vec{a} \times \vec{b})|$

$p = 6|\vec{b} \times \vec{a}| = 6|\vec{a} \times \vec{b}|$

$q = |\vec{a} \times \vec{b}| \Rightarrow \frac{p}{q} = k = 6$

7.



If $\overrightarrow{AB} = -3\hat{i} + 4\hat{k}$

$\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

BM = CM

$AM = |\overrightarrow{AM}| = \left|\frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}\right|$

$= |\hat{i} - \hat{j} + 4\hat{k}| = \sqrt{1+1+16} = \sqrt{18}$

8. $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 3$

$\Rightarrow \vec{a}.\vec{b} = \frac{1}{2}$

Now $(3\vec{a} - 4\vec{b}).(2\vec{a} + 5\vec{b})$

$= 6|\vec{a}|^2 + 15\vec{a}.\vec{b} - 8\vec{a}.\vec{b} - 20|\vec{b}|^2$

$= 6 + \frac{7}{2} - 20 = -\frac{21}{2}$

9. Let the sides of the parallelogram be $\vec{x}$ and $\vec{y}$

Let $\vec{x} + \vec{y} = \vec{a} - \vec{b}$

$\vec{x} - \vec{y} = 3\vec{a} + \vec{b}$

$\therefore \vec{x} = 2\vec{a}$ and $\vec{y} = -(\vec{a} + \vec{b})$

Area of parallelogram

$= |\vec{x} \times \vec{y}|$

$= |-2\vec{a} \times (\vec{a} + \vec{b})|$

$= 2|\vec{a} \times \vec{b}| = 2 . 2 . 1 . \frac{1}{2} = 2$

Alternate

Area of parallelogram $= \frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

$= \frac{1}{2}|4(\vec{a} \times \vec{b})| = 2|\vec{a} \times \vec{b}|$

10. $\vec{a} \cdot (\vec{b} + \vec{c}) = 0$

$\vec{b} \cdot (\vec{c} + \vec{a}) = 0$

$\vec{c} \cdot (\vec{a} + \vec{b}) = 0$

$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$

$|\vec{a} + \vec{b} + \vec{c}| =$

$\sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})}$

$= \sqrt{1+4+9+0} = \sqrt{1+4+9+0} = \sqrt{14}$

11. $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & a & 5 \end{vmatrix} = 0$

$\Rightarrow 3(3 - 2) + a(1 + 6) + 5(4 + 1) = 0$

$\Rightarrow 3 + 7a + 25 = 0 \Rightarrow a = -\frac{28}{7}$

12. $\begin{vmatrix} 1 & 1 & -\lambda \\ 3 & -2 & 4 \\ 3 & -7 & 14 \end{vmatrix} = 0$

13. $\vec{u} + \vec{v} + \vec{w} = 0$

$|\vec{u} + \vec{v} + \vec{w}|^2 = 0$

$|\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}) = 0$

$9 + 16 + 25 + 2(\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}) = 0$

$\therefore (\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}) = -25$

14. $(\vec{a} + \lambda\vec{b}).(\vec{a} - \lambda\vec{b}) = 0$

$\Rightarrow |\vec{a}|^2 = \lambda^2|\vec{b}|^2$

$\Rightarrow \lambda^2 = \frac{1}{9}$ or $|\lambda| = \frac{1}{3}$

15. Volume $= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$

$= 2(4 - 1) - 3(-3 - 2) + 4(-1 - 6)$

$= 6 + 15 - 28 = -7$